

## Tool for initial hull structure dimensioning at ship concept design

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**ABSTRACT** During the ship concept design, the hull weight estimates are generally based on top-down methods such as empirical formulas resulting from statistical analysis of data from existing ships and using as variables the main dimensions and some hull form coefficients. These formulas are generally outdated, and do not reflect the actual configuration of the hull structure. In this work, a tool for the initial dimensioning of the hull structure by class rules was developed as an additional module of a ship numeric model for the ship concept design. A non-linear constrained optimization procedure is used to obtain the scantlings of plates and stiffeners that minimize a measure of the structural weight per unit length. The methodology adopted and the data models are presented as well as the results of a case study application.

### 1 INTRODUCTION

During the concept stage of ship design, due to the reduced amount of information available, a number of tasks are carried out in a simplified way and using mostly data estimated from empirical methods. This is the case of the hull structure weight that is generally estimated by some empirical expression as a function of the ship's length, breadth, depth and block coefficient (Schneekluth and Bertram, 1998). In merchant ships, the hull weight accounts for about 70% of the lightship weight, and in ships with the engine room located aft, the weight of the cargo area can sum up to 70% of the total hull weight.

Taking this into consideration, is developed a concept that allows to improve the estimation of the weight of the cargo area by producing a midship section configuration (considering plates and stiffeners) and estimating the total weight as function of it. In order to achieve this, many considerations and limitations have to be taken into account, as the produced structural configuration at midship shall be realistic (e.g. few standard elements), feasible (considered economically and for production) and compliant with the rule sets defined by the classification societies.

In this work is developed a tool for the initial dimensioning of the scantlings in the midship section of a steel hull, which, through an optimization procedure, allows to comply with specified requirements. Although in the tool this optimization procedure is applied exclusively on longitudinal elements, in a second stage also the transversal elements are taken into consideration, as their specifications depend on the main dimensions and

their effect over the total structural weight can't be neglected to achieve an accurate estimation.

Objective of the presented tool is to define a feasible structural configuration of the considered section based on few initial data (mostly main dimensions and ship design parameters), which in a quick but still accurate way, estimates the section modulus and the weight per unit length of the cargo area with more precision as using only empirical methods. Thus, the challenge is to provide a procedure that allows to define the structural configuration of a cross-section in the early stage of a conceptual design when main dimensions are known, and the structural arrangement has to be estimated. Having as input the main dimensions, the task is to create a cross-section typical for the specified ship type, defining a feasible plates and stiffeners arrangement that fulfills the requirements imposed by the classification societies over the local and global minimum section modulus and on the minimum thickness of the structural elements.

A number of parameters allows the user to define the profile type of the stiffeners, the maximum width of plates, height of double bottom and width of the side tanks. Moreover, the algorithm used considers the midship section formed by stiffened panels, structural elements that define a number of plates and stiffeners with constant dimensions, material, stiffener's profile and spacing. This concept allows to choose in a flexible way the most convenient arrangement of structural components, but still being realistic regarding the variety of the elements.

The concept consents to dimension the structural configuration based on the typical arrangement for the considered ship type with symmetric

midship section. In the presented tool the case study is a multi-purpose/container vessel of small dimensions (feeder ship).

## 2 STATE OF THE ART

Over the years the structure of a vessel has been analyzed with the objective to get more precise estimations and eventually to propose improvements to the ship design process by optimizing the total weight or costs of the vessel.

Rigo (2001a, 2001b, 2003) presented an optimization procedure for the ship hull to be used in the preliminary design, in which the scantlings are computed based on first principles and the minimizations of the weight and of the normalized building cost were used as objectives, either individually or combined. The design variables are the thicknesses of the plates. For the longitudinal stiffeners and the transverse frames the variables are the spacing, the height and thickness of the web and the width of the flange.

Andrews (2006) applied the computer-aided environment on the preliminary ship design stage on single structural modules, in order to analyze and compare outputs deriving from different solutions, being in this way possible to evaluate different solutions and produce a more detailed preliminary design.

Ehlers et al. (2010) presented an optimization procedure for the concept design of the structure of a chemical product tanker. Only the longitudinal structure is considered for the ship weight, hull production cost evaluation and its fatigue life. The analyzed tanker structure is composed by one corrugated bulkhead and 22 stiffened panels. The bulkhead is defined by the corrugation geometry and the plate thickness. Each of the stiffened panels is defined by three design variables: plate thickness, type and number of stiffeners.

Sun and Wang (2012) applied on the midship section of a VLCC a modelling procedure combined with a GA-based (Genetic Algorithm) optimization procedure, which has a set of 35 design variables relative to the plates thickness and beams dimensions. As the design variables are related exclusively to longitudinal elements, the chosen objective function is the minimization of the mass of all longitudinal structures in the central body of the VLCC.

Ma et al. (2014) also aimed the optimization of the scantlings in the cross section by applying the finite element method on every plate of the structure, analyzed individually. It was adopted a GA-based multi-objective optimization to obtain the minimum weight and costs, and the maximum value for the safety measure.

Andric et al. (2017) proposed a multi-objective scantling optimization applied on the structure of a cruise vessel with the aim to minimize the structural weight and the vertical center of gravity, having as design variable the scantling of the plates. The set of optimal solutions has been further analyzed with a Pareto frontier.

Gaspar et al. (2012) decompose the structure in subcomponents with defined characteristics, to reduce the complexity of the problem and at the same time producing information about the structural subcategory, which is more detailed and accurate. Similarly, the structural module taken into consideration for this tool is the midship section, that is the most relevant section in order to estimate weights, structural strength and costs of the total vessel.

The previous work done shows how through different methods is aimed the optimization of weights and costs through the modification of selected design variables. While Rigo (2001a) optimizes weights and costs adopting as design variables the plate thickness, number of stiffeners, stiffeners' profile and spacing, Ehlers et al. (2010) define a model where the spacing is not taken into consideration, but the variables are associated to one stiffened panel in order to get minimum structure weight, cost and fatigue life. Ehlers (2010) applies the same procedure over the side structure of the hull in order to define the most convenient structural configuration in function of the crash-worthiness, having as design variables the plate thickness, stiffener types and spacing.

Being the objective of the presented approach to define initial hull structure scantlings through an optimization process, rather than the optimization of such scantlings, it is aimed just the minimization of the weights of the structure (that is important to increase service speed, cargo carried or to minimize the building costs), having as design variables the thickness of the plates in the deck, from which depend respective stiffeners dimensions and spacing of longitudinal and transversal elements.

In recent years the interest over a software that could support the engineer in the design process has grown, and some of the classification societies have developed software tools able to perform different levels of strength assessment based on a variable number of inputs. For example, Lloyd's Register and Bureau Veritas have developed respectively the software "RulesCalc" and "MARS2000", which are able to perform an analysis over a structure given by the user and check the rules compliance. In addition to rules compliance checking, the "Leonardo" system by RINA implements a FE analysis of the given structure. DNV-GL developed the "Poseidon" system, which also allows to perform an automatic initial scantling determination.

### 3 MIDSHIP SECTION LONGITUDINAL STRUCTURE

In this work the hull structure is represented by 2D representations of cross sections in the cargo area. The procedure developed is a compromise between accuracy and the level of detail. A number of simplifications were assumed to reduce the computational time so that it can be included in the ship optimization process. The geometry of the hull molded lines is described by a number of cross sections represented by polylines. The scantlings of plates and stiffeners are determined based on the requirements of a classification society (DNV GL, 2016; DNV GL, 2016a).

Main objective of the work is to define the structural configuration and the initial dimensioning of the scantlings in the selected section and to perform over these a strength, economic and reliability analysis. In the algorithm presented in Figure 1 are used as inputs the vessel's main dimensions, ship type and, if specified by the user, hull compartment parameters and specific shipyard's constraints.

#### 3.1 Methodology

Firstly, the geometry of the section and the structural configuration are defined in accordance with the global parameters associated with the specific ship type Figure 1. The dimensions that are used as input in the tool are ship's main dimensions ( $L_{pp}$ ,  $B$ ,  $D$ ,  $T$ ,  $C_b$  and  $C_m$ ), structural specifications (maximum plate width and minimum structural thickness) and the hull compartment parameters (double bottom height and side tanks width), which can be defined by the user out of a defined range of dimensions depending on the ship type.

The continuous longitudinal material is then decomposed in stiffened panels (*Bottom*, *Bilge*, *Side*, *Deck*, *DoubleBottom*, *BottomGirders*, *LongitudinalBulkhead*, *SideGirders* and *Hatch*) with defined plates and stiffeners.

For this purpose, for each panel is specified a *PlateSet* and a *StiffenersSet*. A *PlateSet* is an array of groups of identical plates, specifying for each of them quantity, width and thickness. A *StiffenerSet* is an array of identical stiffeners belonging to the same plate, specifying for each a quantity, the profile section type, the orientation, the dimensions and the spacing.

In order to define the scantling for each of the plate sets, rules of the DNV-GL classification society are taken into consideration, which constraint the minimum thickness based on two methods. The chosen thickness is the highest of the values resulting from the two methods. Other rules of the same classification society are also applied to calculate the local minimum section modulus respective to

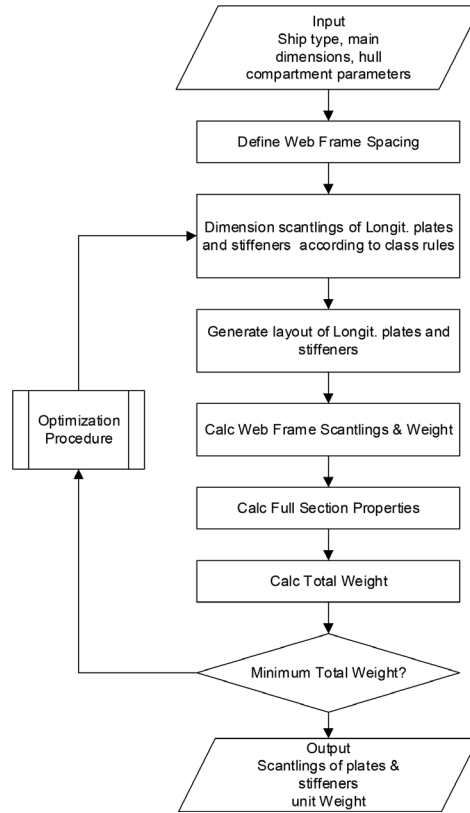


Figure 1. Flow diagram of the process.

a specific stiffener and its minimum web thickness (that takes also into account the ratio between web of stiffeners and thickness of the plate at which the stiffener is attached to). In this way, having defined as default the profile section type of the stiffeners for each of the stiffened panels, it is possible to consult the embedded catalog of standardized stiffener profiles and chose the one with the lowest sectional area that complies with minimum local section modulus and minimum web thickness.

Rules of the classification society are also applied on the whole structure, to define the associated wave coefficient ( $C_w$ ) and the minimum section modulus relative to deck and to bottom, which will be used in the optimization tool as constraint.

As the midship section structure needs to be computed geometrically in order to be evaluated, polylines describing the shape are assigned to the stiffened panels. Both the elements stored in the stiffened panel (plates and stiffeners) are in this way associated to a position. With this process it is possible to obtain the complete structure layout. The number of stiffeners associated to each panel is obtained by dividing the available space by the stiffeners spacing,

similar approach adopted to get the number of vertical girders in the double bottom and of horizontal girders in the side tanks, where the available space is divided by the girder spacing, in this case chosen to be three times the stiffener spacing.

Stiffeners and girders are initially created independently from each other, so that these elements can be equally distributed along the structure. In a second stage the position of the stiffeners is checked again, to erase the stiffeners that are too close to a girder or to another stiffener, as it would be impossible to weld these (chosen tolerance for this purpose is 0.1 m). Similarly, also plates dimensions need to be constrained, by avoiding designing plates with width smaller than 1 m (with exception of the sheer strake) and bigger than the maximum width imposed at the beginning of the tool (2.5 m, with exception of the bilge plate), with a thickness not smaller than 7 mm, imposed as the minimum feasible scantling for strength requirements.

In the horizontal stiffened panels (*Bottom, DoubleBottom, Deck*) the stiffeners are located starting from the longitudinal center plane to the sides in order to maintain the vertical alignment even when the hull shape changes.

Being at this point the sectional structure fully described, it is possible to compute geometrical properties like the sectional area, moments of inertia, neutral axis and resulting section modulus. Having sectional area and material properties, the longitudinal contribution to the weight per unit length of the parallel body of the vessel can be calculated.

The produced midship section is limited to symmetric shapes, as just half of the section is designed and mirrored to create the complete section. This tool is designed with the objective to analyze multipurpose/container vessels with typical dimensions of a feeder ship.

### 3.2 Data model

The methodology described above was implemented in an object-oriented code whose main classes are presented in Figure 2.

The central class *CrossSection*, the basis of the data structure, contains the description of a single selected section of the vessel, which is associated to ship characteristics through the class *Ship* (main dimensions, type, etc.) and for which rules of the classification society are computed in *ClassRules*, in order to establish constraints regarding the design of the structure.

The class of *StiffenedPanel* consists in the description of panels that form the whole cross-section of the vessel. A *StiffenedPanel* is composed by a set of plates welded side by side and reinforced by stiffeners. It is assumed that within a panel the thickness of the plates and the profile of the stiff-

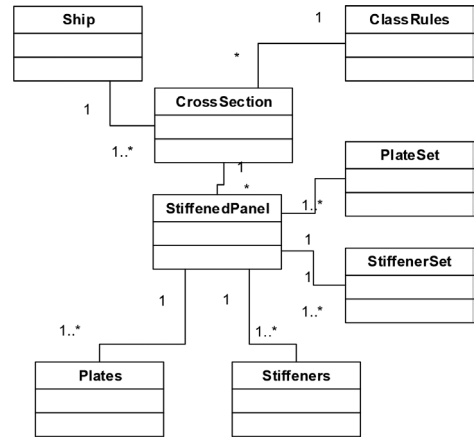


Figure 2. Main data classes developed for the procedure.

eners are constant, reason for which the classes *PlateSet* and *StiffenerSet* are adopted, in order to store the qualitative description of the element type relative to a panel. The *PlateSet* stores information about the quantity, width, thickness and material of a group of plates, the *StiffenerSet* class has information about the quantity, section profile type and dimensions, spacing, web and flange direction and material of stiffeners associated to a panel.

While the classes *PlateSet* and *StiffenerSet* are associated to a group of elements, the classes *Plates* and *Stiffeners* are the actual description of these elements, being associated individually to each of these.

After that the required information is stored in the class *CrossSection*, it is possible to calculate the weight per unit length based on the analyzed section, being given as objective function of the tool.

## 4 TRANSVERSE STRUCTURE

In this work the transverse structure is not optimized, being dimensioned in accordance to the classification society rules, in order to complement the longitudinal weight and obtain in this way the total unit weight estimate.

A single typical web frame in the midship section area is dimensioned both for the double-bottom and the side-shell regions. The weight of the complete frame is then computed and divided by the web frame spacing as a contribution to the hull weight per unit length.

The dimensioning of the typical transverse bulkhead is not yet considered in this work. The contribution of transversal material to the total weight is obtained by multiplying the total number of bulkheads in the cargo area by the individual

weight which is computed by multiplying the cross-sectional area by an empirical index in ton/m<sup>2</sup>.

## 5 OPTIMIZATION PROCEDURE

In this tool the optimization process is adopted with the aim to find a feasible solution for the initial dimensioning of the stiffened panels in the midship cross section. This solution is not to be intended as the best structural configuration for the given section as the optimization algorithm is applied exclusively to guarantee the compliance with selected requirements, as described in section 5.2.

The optimization procedure adopted is based on the genetic algorithm, but the convergence criteria has not been applied as, as said, objective of the tool is to produce feasible outputs.

### 5.1 Design variables

The design variables for the model of the multi-purpose/container vessel are the thicknesses of the plates in the panels in the upper part of the section, as being more distant from the neutral axis have a more critical and sensitive role in the compliance to the overall minimum section modulus imposed by the classification societies. Indeed, the upper part of the structure in such type of vessels presents a section modulus that is lower than the one in the lower part, as the material in there is less and thus the neutral axis is lower. The analyzed thicknesses are identified with the upper longitudinal bulkhead panel and the deck panel. The selection of the design variables is limited to the upper panels as these have the biggest influence over the section modulus. Moreover, the number of these is chosen to be low as, having more design variables, would lead to higher computational time without improving the output quality.

The design variables are assumed to have an initial value equal to the minimum local scantling defined by the classification societies, also considered to be the lower bound of the variable during the optimization process.

### 5.2 Constraints

The bounds applied over the design variables used for the single-objective genetic algorithm, as described in the previous chapter, are the minimum thickness imposed by the classification society as lower bound and the double of such value as upper bound:

$$t_{CSR} \leq t_i \leq 2 * t_{CSR}$$

where  $t_{CSR}$  is the minimum thickness for a specified plate and  $t_i$  is the actual plate thickness.

As the algorithm adopted for the optimization process does not deal with discrete values and thus it is not possible to create a catalog of available plate thicknesses, the obtained feasible distribution of the scantlings has been rounded up to the next integer number. This process shall be improved in a second moment by adopting a different algorithm, which enables to select the plate scantling from a given catalog specific to the shipyard.

Constraint applied over the optimization process is the section modulus at deck and bottom that must be bigger than the minimum section modulus defined by the classification societies:

$$\begin{aligned} W_{bottom} &\geq W_{min,CSR} \\ W_{deck} &\geq W_{min,CSR} \end{aligned}$$

where  $W_{bottom}$  and  $W_{deck}$  are the calculated section moduli relative to the bottom and deck, and  $W_{min,CSR}$  is the minimum section modulus imposed by the classification societies.

Other types of constraints applied in the tool are indirect, as they are not part of the optimization process, but the algorithm of the tool constraints the possible range of application. This is the case of stiffener selection, which is done by computing the required local section modulus as given by the classification societies and choosing out of a catalog the stiffener with the lowest sectional area that complies with such requirement. Another constraint considered for the selection of stiffeners is the minimum web thickness, which must be higher than 90% of the scantling of the plate to which the stiffener is attached. Output from the stiffener selection is the stiffener type and its web and, eventually, flange dimensions.

### 5.3 Objective function

Although Winkle & Baird (1986) investigated the fabrication cost of stiffened ship structures and came to the conclusion that relative cost optima rarely bear any relation to traditional minimum weight criteria, being the aim of the tool to find a quick and feasible solution, it is enough to minimize the weight per unit length. Moreover, the objective of the presented algorithm is to estimate initial dimensions for the structural configuration through an iteration procedure defined by the optimization process, rather than to find the minimum for weights or costs. The total unit weight is calculated as follows:

$$W_{tot} = A_{section} \gamma_{steel} + (V_{web} \gamma_{steel}) / S_{web}$$

where  $W_{tot}$  is the total unit weight in [t/m],  $A_{section}$  is the area of the longitudinal structural components,  $\gamma_{steel}$  is the specific weight of steel assumed to

be  $7.85 \text{ [t/m}^3\text{]}$ ,  $V_{web}$  is the volume of the transverse structural elements in one frame and  $S_{web}$  is the web frame spacing.

In order to keep the initial dimensioning process fast, the total weight is split into longitudinal and transversal components, where the first depends on the optimization process, and the second on the stiffeners spacing, which allows the program to be fast, at the expense of the level of detail of the transverse components.

#### 5.4 Optimization algorithm

A non-linear constrained single-objective algorithm is adopted. The algorithm aims the minimization of the objective function, the weight per unit length, by increasing or decreasing the variables associated to the scantling of the plates in the upper part of the external side shell, in the upper part of the longitudinal bulkhead, used as double side shell, and in the deck.

## 6 RESULTS AND VALIDATION

The design of a small multi-purpose ship is adopted as a case study for the developed procedure.

Main dimensions of the considered vessel are shown in Table 1.

In addition to these values, the double bottom height is assumed to be 1.2 m, the side tank width to be 1.6 m, the maximum plate width to be 2.5 m, to comply with requirements of the shipyard, and the overall minimum plate thickness is assumed to be 7 mm.

For this case are chosen the flat bar stiffeners in the bottom, the double bottom and girders' panels, while in the side, longitudinal bulkhead and deck panels are adopted stiffeners with the bulb profile.

The produced midship section is shown in Figure 3 and consists of a set of panels with associated plates and stiffeners. The section before the optimization presents values and characteristics as shown in Table.

As it can be seen in the previous table, the values produced from the given initial dimensions deliver

a section modulus at the bottom that does not comply with the requirements imposed on the minimum section modulus by the classification society.

The tool, in order to verify the imposed constraints on the section modulus, modifies the scantling of the plates in the mentioned panels until all the requirements are fulfilled. After the optimization process is concluded, the constraints are verified and a local minimum for the weight is found, the tool presents as output the values listed in Table 2.

The final arrangement of the midship section consists of a total of 22 plates (with scantling range between 7 mm and 18 mm) and 45 longitudinal stiffeners, subdivided in 24 with a flat bar profile and 21 with bulb profile.

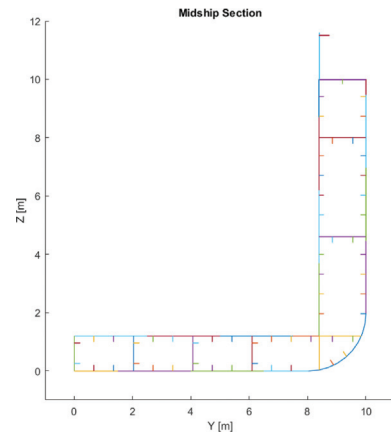


Figure 3. Produced midship section.

Table 2. Output values before optimization.

Definition	Value	Unit
Total sect. area	1.1	m <sup>2</sup>
Neutral axis	3.16	m
Unit weight	9.0	t/m
Sect. modulus deck	4.534	m <sup>3</sup>
Sect. modulus bottom	2.095	m <sup>3</sup>
Min. sect. modulus	3.09	m <sup>3</sup>

Table 1. Main dimensions of the considered example.

	Value	Unit
$L_{pp}$	115.0	m
$B$	20.00	m
$D$	10.00	m
$T$	8.30	m
$C_b$	0.72	
$C_m$	0.99	

Table 3. Output value after optimization.

Definition	Value	Unit
Total sect. area	1.3	m <sup>2</sup>
Neutral axis	3.83	m
Unit weight	10.3	t/m
Sect. modulus deck	4.973	m <sup>3</sup>
Sect. modulus bottom	3.09	m <sup>3</sup>
Min. sect. modulus	3.09	m <sup>3</sup>

## 7 CONCLUSIONS

The concepts and the methodology developed for the task of the initial structural dimensioning of the hull were presented. This methodology was implemented as a computational tool to be used as a new module of a system for the determination of the ship main characteristics at the early concept design stage.

This concept has been driven by the need of a practical and fast algorithm to provide an initial structural arrangement of the cross-section, to analyze the influence of different design factors on ship design, to allow the user to check what is the effect of different structural configurations on the unit weight and to check its feasibility. Different structural configurations can be obtained by changing stiffeners spacing and profile, double bottom height, width of side tanks, number of bottom and side girders. Main challenge of this procedure is to define an algorithm which estimates the initial structural arrangement of the typical cross-section and is adaptable to different ship types and ship dimensions.

The work presents a method to design a feasible structure configuration, where the scantlings in the defined section are obtained through an optimization procedure. The single-objective optimization procedure is carried minimizing the weight, having as design variables the scantlings of the panels in the upper part of the section (deck and upper plates of side and bulkhead), which allows the program to be fast and to present a feasible arrangement that complies with classification societies rules. The algorithm is able to select the stiffener dimensions out of an imbedded catalog with standard elements, to ensure that variety of components is limited. Same procedure shall be implemented in the future over the plate thickness, with an imbedded catalog of available standard thicknesses, which, up to this point, has been approximated by rounding up the value obtained from the initial dimensioning.

Profile type of the stiffeners, maximum width of plates, height of double bottom and width of the side tanks can be chosen by the user, in order to create a case-specific model that takes into account the shipyard specifications and shipowner preferences, besides the structural configuration specific to the ship type. Moreover, the section is formed by panels, which are structural elements grouping plates and stiffeners with constant dimensions, material, stiffener's type and spacing. This tool structure allows a flexible selection of the most convenient arrangement of structural components, but still being realistic regarding the variety of the elements, as the properties are associated to the stiffened panels.

Although this process has general applicability independent from ship type or dimensions, in this

work is shown the case applied on a feeder multi-purpose/container vessel.

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