Dependence of Ultimate Bending Moment of Box Girders on Panel's Slenderness

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Abstract: The structural behavior of box-girders under pure bending moment is analyzed and discussed on the basis of the results of experiments performed by the author. The box girders are geometrically similar but made of different materials and having different stiffener's geometries. The influence of the main parameters that influence the ultimate strength of box-girders under pure bending moment is analyzed and discussed. Practical design formulas are derived and presented, allowing for a fast evaluation of the performance of the boxes under pure bending. The achievements may be extrapolated to the analysis of the hull girder of ship and used as a basis for the structural codes of ship design.

Key words: Box-girder; Ultimate bending moment; Stiffened panel; Ultimate strength; Buckling

INTRODUCTION

The determination of ultimate strength of thin-walled structures under bending moment is a very important issue on the analysis of the structural performance of ship's hull girder and the safety of the ships. Due to its similar geometry, a box-girder is commonly accepted as representative of such behavior and several researchers used boxes as basis to performed pure bending tests instead of scaled models of ship's structure (Dowling et al., 1973; Gordo & Guedes Soares, 2009, 2013, 2015, 2018; Nishihara, 1984; Saad-Eldeen et al., 2011, 2013).

The purpose of the present study is to derive simplified expressions allowing to estimate the ultimate bending moment capacity of thin-walled structures taking in to consideration the main parameters that affect the structural strength. To fullfil such task, the results of previous tests are used.

FUNDAMENTALS

The hull-girder of a ship behaves like a Euler's beam when subject to a bending moment distribution along its length. Thus, the distribution of stresses $\sigma(x)$ on a cross-section depends mostly on the applied bending moment *M*, the inertial moment of area *I* in relation to neutral axis and the distance of point into consideration to that axis *z*, and may be expressed in the linear elastic range by eq. (1).

$$\sigma(x) = \frac{M}{I} z \tag{1}$$

However real structures have initial imperfections which cause non-linear behavior and, more important, thin-walled structures suffer from reduction of effectiveness with increasing loading due to buckling. These two issues make the moment-curvature response of the structure very non-linear and reduce normally the ultimate carrying capacity of the hull under bending.

Initial imperfections are important on the parts of the structure in compression because they may promote premature local or global buckling. Those in tension are not important because the amplitude reduces with loading.

The amplitude of initial imperfections is dependent on the plate's slenderness β (Kmiecik et al., 1995).

$$\beta = \frac{b}{t}\sqrt{\frac{S_{yp}}{E}} \tag{2}$$

b and *t* are the width and the thickness of the plate element between stiffeners; S_{yp} and E are the yield stress and Young's modulus of the material.

Buckling of stiffened panels under axial compression occurs on 1 of 3 different types or a combination of them: plate buckling, column buckling and tripping. The ultimate axial strength of plates of the first type depends mostly on β, and the 2 others are dependent on the structural behavior of the stiffener with associated plating which is characterized mainly by the column's slenderness λ.

$$\lambda = \frac{a}{r}\sqrt{\frac{\sigma_o}{E}} \tag{3}$$

The span between transversal stiffeners is *a* and *r* is the radius of gyration of stiffener and associated plate cross-section around the its neutral axis with a second moment of area I_s and defined as:

$$r = \sqrt{\frac{I_s}{A_t}} \tag{4}$$

A_t is the sectional area of the stiffener and associated plating, composed by A_p and A_s that designates respectively the cross-section area of the associated plate and of the stiffener.

So, the prediction of the ultimate bending moment (*UBM*) of thin-walled structures should be function of the slenderness parameters β and λ.

EXPERIMENTAL DATA

The study uses as database the results of 6 similar tests performed on mild steel box-girders with almost identical geometry.

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Three of them belong to the same series, denotes as M series and they have different plate thickness, respectively 4, 3 and 2 mm plate's thickness (Gordo, 2002; Gordo & Guedes Soares, 2004, 2015, 2018); the others 3 belong to a different series (N series) with more than one frame' span and small stiffener' spacing (150mm) but with similar cross section arrangement (Gordo & Guedes Soares, 2009, 2013).

The tests are performed by applying symmetrical 4 loading points which induces pure bending in central part of the structure to be tested. Figure 1 shows the setup of typical test and Figure 2 presents its schematic diagram.

Table 1 presents the geometrical properties of the box-girder and the mechanical characteristics of the material.

The results of the tests are presented in Table 2 where it is calculated the structural efficiency (SE) given by ratio between the ultimate bending moment (UBM) and the first yield bending moment (YBM) and the ratio between the UBM and the structural modulus (EI) or a measure of the section modulus assumed as EI/D for objectivity. D is the nominal height of the box-girder. The geometry of the bar stiffeners is given by their height *h* and thickness t_w. A_b is total cross-section area of the box-girder and R its radii of gyration.

Figure 1 Setup of experiment of M series in location

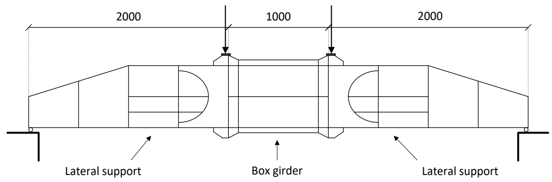

Figure 2 Schematic diagram of M series test

Table 1 Geometrical and mechanical properties of boxes

	M4-200	**M3-200**	**M2-200**	**N200**	**N300**	**N400**
a (mm)	800	800	800	200	300	400
t (mm)	4.1	3	2	4	4	4
b (mm)	200	200	200	150	150	150
b/t	48.8	66.7	100.	37.5	37.5	37.5
S_{yp} (MPa)	310	183	177	270	270	270
S_{ys}(MPa)	240	310	183	270	270	270
E (GPa)	210	210	210	200	200	200
I (dm^4)	8.33	6.86	4.13	7.68	7.68	7.68
A_b (dm^2)	1.13	0.98	0.63	1.21	1.21	1.21
R (mm)	272	264	256	252	252	252
2R/D	0.91	0.88	0.85	0.84	0.84	0.84
h (mm)	45	45	30	20	20	20
t_w (mm)	6	4	3	4	4	4
A_t (mm^2)	1090	780	490	680	680	680
A_p (mm^2)	820	600	400	600	600	600
A_s (mm^2)	270	180	90	80	80	80
r (mm)	8.6	8.6	8.6	7.6	7.6	7.6
a/r	93	93	93	26	40	53
β	**1.87**	**1.97**	**2.90**	**1.38**	**1.38**	**1.38**
λ	**2.98**	**2.17**	**3.05**	**0.97**	**1.46**	**1.94**

Table 2 Results of tests and calculations

	M4-200	M3-200	M2-200	N200	N300	N400
EI (MNm^2)	215	151	87	154	154	154
Yield Bending Moment (kNm)	890	419	244	669	669	669
Ultimate Bending Moment (kNm)	609	349	173	643	512	475
SE - Structural Efficiency	0.68	0.83	0.71	0.96	0.77	0.71
UBM/EI (1/(1000.m))	2.83	2.31	1.99	4.19	3.33	3.09
UBM*D/EI (1/(1000)	1.70	1.39	1.20	2.51	2.00	1.86

EFFECT OF SLENDERNESS

As the stiffened plate elements in compression lose part of their axial strength due to buckling and initial imperfections, the ultimate strength should be affected by the variation on the slenderness parameters β and λ. This dependence may be expressed as:

$$UBM = \varphi(\beta, \lambda) \cdot EI \qquad (5)$$

$\varphi(\beta, \lambda)$ represents that loss of strength but its effect is less than the reduction in strength on the stiffened elements alone because only part of the box-girder structure is under high levels of compression. The parts under tension are fully effective and their contribution to the overall strength of the box under bending is only affected by shift of the neutral axis during loading (Gordo et al., 1996).

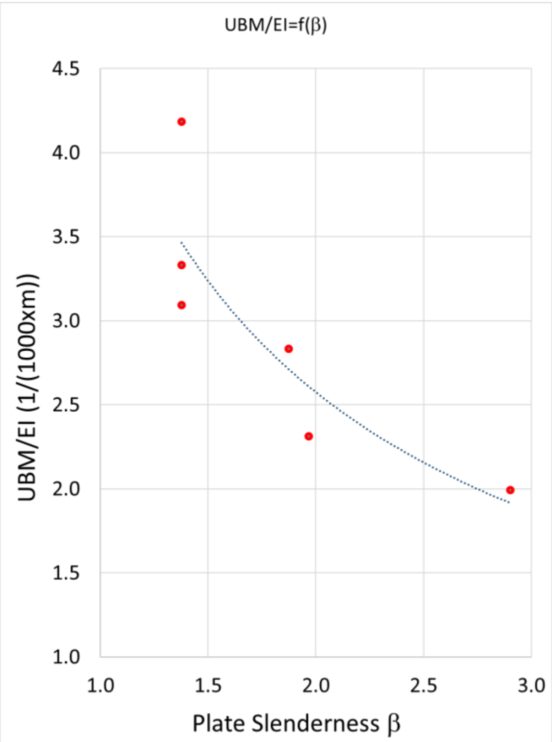

Figure 3 Relationship between UBM/EI and the plate slenderness β

Figure 3 presents the relationship between the UBM/EI and the plate slenderness β. This ratio (UBM/EI) is typically a unitary bending moment in relation to the geometry of the cross-section in terms of dimensions and thicknesses and should be a measure to compare different types of box-girders and ships made of different materials.

Figure 4 plots the dependency of the same quantity in relation to the column slenderness. Here the effect is more marked and, at least according to these data, more important than the effect of the plate slenderness.

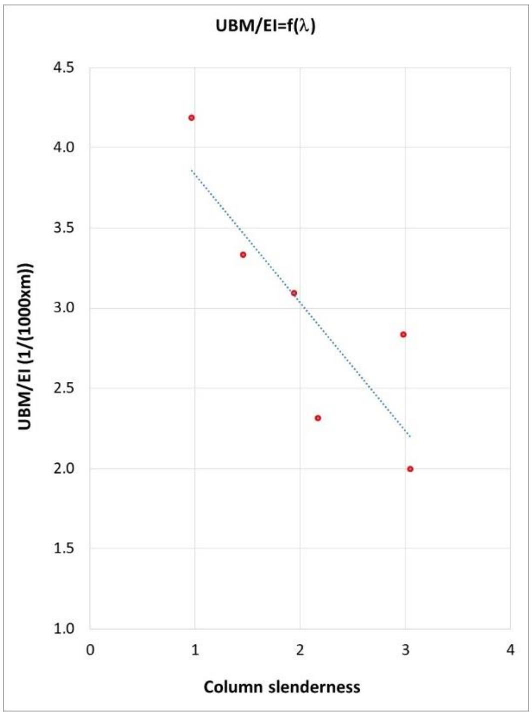

Figure 4 Relationship between UBM/EI and the column slenderness λ

A trial has been made to compute the effect of both parameters together, simply by multiplying them. The dependency is almost linear when the inverse of the product of both plate and column slenderness is considered. The results are plotted in Figure 5. The UBM may be expressed as:

$$UBM = \left(1.84 + \frac{3.13}{\beta\lambda}\right) EI \cdot 10^{-3} \qquad (6)$$

Finally, it should be said that the ratio UBM/(EI/D) should be more representative for futures analyses where different geometries are compared but it was no effect on this data since *D*, the box-girder depth, is the same for all boxes. It results for this data in:

$$UBM = \left(5.21 + \frac{3.07}{\beta\lambda}\right)\frac{EI}{D+2h} \cdot 10^{-3} \qquad (7)$$

Figure presents this relationship that includes the small corrections due to the differences on the height of the web of the stiffeners.

Both formula use IS unit system.

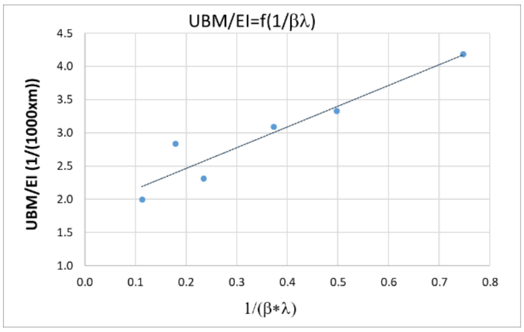

Figure 5 Relationship between UBM/EI and the product of slenderness β.λ

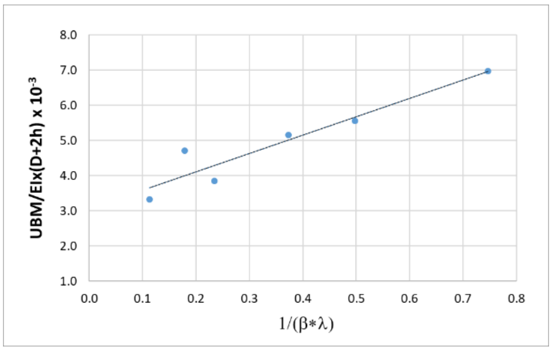

Figure 6 Relationship between (UBM/EI).(D+2h) and the inverse of the product of slenderness β.λ

EFFECT OF YIELD STRESS OF THE MATERIAL

Instead of using the structural modulus EI as reference, it can be used the product of the material's yield stress and the inertial moment of area, $S_{yp}I$. The main difficulty arises in relation to the value to be used for S_{yp} because, often, the associated plate and the stiffeners have different properties. This problem is addressed by Gordo & Guedes Soares (1996) and may be solved partially by using the concept of equivalent yield stress. In this section the plating's yield stress is used.

Figure 7 shows the best relationship found between the normalized bending moment and the inverse of βλ.

The equation of the regression line is given by:

$$UBM = \left(1.34 + \frac{0.54}{\beta\lambda}\right)\frac{S_{yp}I}{D+2h} \qquad (8)$$

However, the tendency is not very marked (R^2=0.45) and more data is required using high tensile steel for clarification.

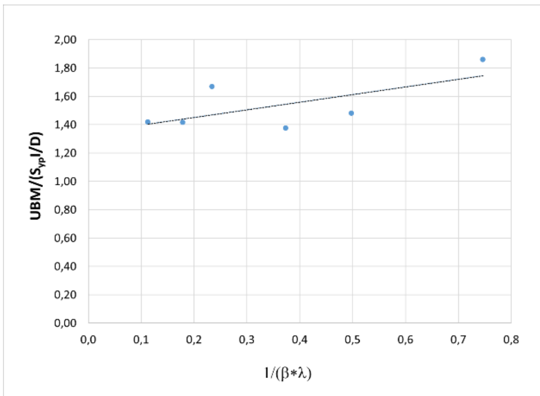

Figure 7 Relationship between $(UBM/S_{yp}I).(D+2h)$ and the inverse of product of slenderness β.λ

CONCLUSIONS

The dependence of the ultimate bending moment of box-girders on plate's and column's slenderness is accessed and formulas are presented which may serve as basis for integration on structural codes for ship structures design.

These 2 parameters reduce the efficiency of the of the box girder when they increase, or in other words, with a slenderer structure.

The effect can be computed independently for each parameter but a linear relation is very marked with the inverse of the product of β and λ.

The analysis of the effect of the yield stress of the material is not conclusive due to a low variation of this parameter on the database.

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